Scaling Relations for Auxin Waves

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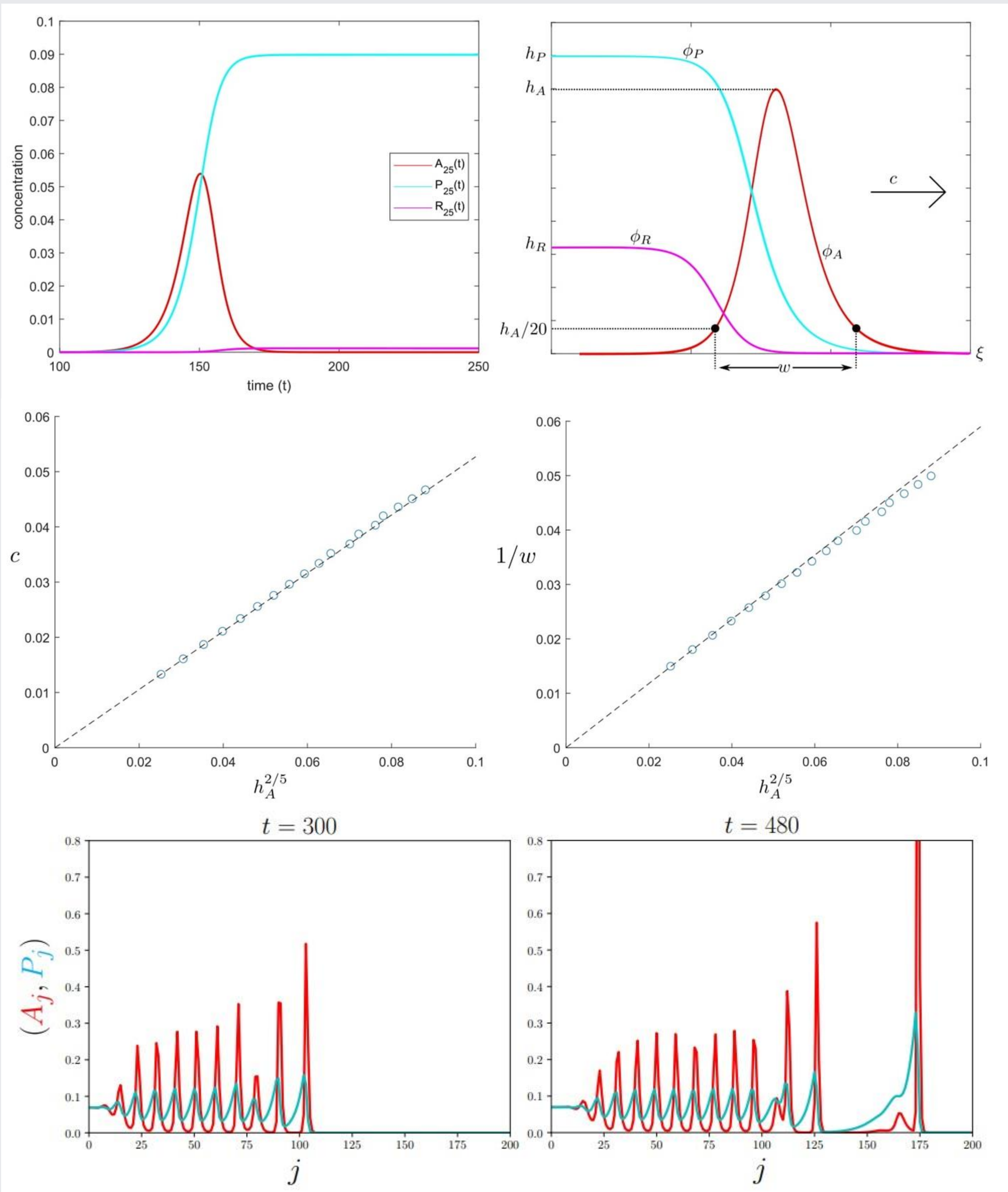
Auxin Transport

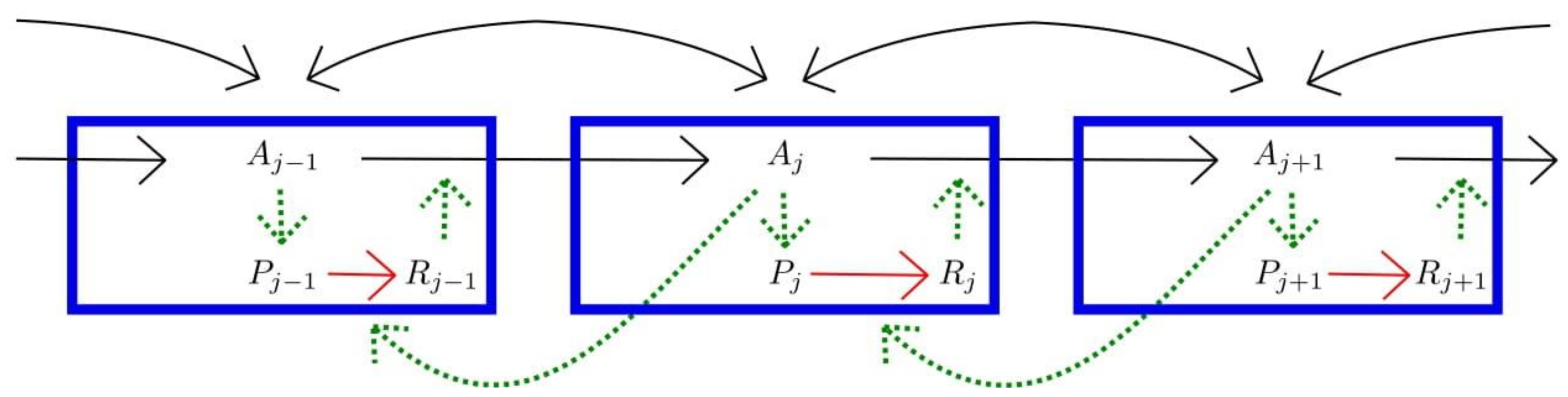
- The phytohormone auxin (A) plays a crucial role in the development and the growth of plants.[1]
- Auxin transport between plant cells is facilitated by the PIN protein, which may be polarized (R) or unpolarized (P).
- The PIN polarization rate within a cell is influenced by the auxin concentration in adjacent cells.
- We study phytohormone transport over an infinite, one-dimensional "row" of cells.[2]

Notation

- The cells in the row are indexed by integers $j \in \mathbb{Z}$.
- $A_j(t)$ = auxin concentration in cell j at time t.
- $P_j(t)$ = unpolarized PIN concentration in cell j at time t.
- $R_j(t)$ = right-polarized PIN concentration in cell j at time t.



The Model[3]



$$\begin{cases} \dot{A}_j = T_{\text{act}}\left(R_{j-1}\dfrac{A_{j-1}}{k_a + A_{j-1}} - R_j\dfrac{A_j}{k_a + A_j}\right) + T_{\text{diff}}(A_{j+1} - 2A_j + A_{j-1}), \\ \dot{P}_j = -k_1\dfrac{A_{j+1}}{k_r + A_{j+1}}\left(\dfrac{P_j}{k_m + P_j}\right) + \alpha A_j + k_2 R_j, \\ \dot{R}_j = k_1\dfrac{A_{j+1}}{k_r + A_{j+1}}\left(\dfrac{P_j}{k_m + P_j}\right) - k_2 R_j. \end{cases}$$

Numerical Exploration

- The simulations suggest that a pulse of auxin moves through the cells from left to right.
- Before the pulse, all concentrations are very low. After the pulse, for $k_2 = 0$, the PIN concentrations reach constant values.
- The graphs of successive A_j, P_j and R_j appear to be the same graph, just translated to the right by the cell index j.
- We therefore make a traveling wave ansatz: there is a profile function φ and a wave speed $c \in \mathbb{R}$ such that $A_j(t) = \varphi_A(j - ct)$, $P_j(t) = \varphi_P(j - ct)$ and $R_j(t) = \varphi_R(j - ct)$.
- There are clear relations among the auxin wave height h_A $(= \epsilon)$, the auxin wave width w, and the numerical wave speed c:

$$c = c_0 h_A^{\frac{2}{5}} \qquad \text{and} \qquad w \sim \frac{1}{h_A^{2/5}}$$

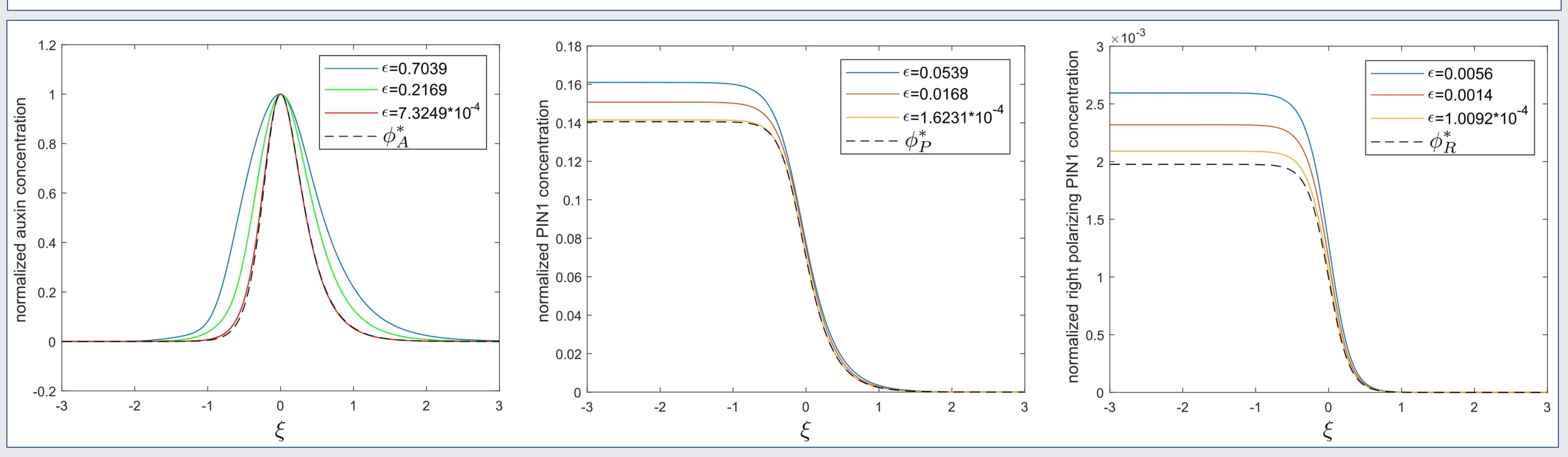
- For a persistent auxin source, the model admits wavetrain solutions.
- Higher pulses travel faster than lower pulses, in correspondence with the scaling relations.

Main Theorem

There exists $\epsilon_* > 0$ such that for $0 < \epsilon < \epsilon_*$ there are functions $\phi_A^\epsilon, \phi_P^\epsilon$ and ϕ_R^ϵ such that taking

$$A_j(t) = \epsilon\phi_A^*(\epsilon^{2/5}(j - \epsilon^{2/5}c_0 t)) + \epsilon^{17/15}\phi_A^\epsilon(\epsilon^{2/5}(j - \epsilon^{2/5}c_0 t)),$$
$$P_j(t) = \epsilon^{1/5}\phi_P^*(\epsilon^{2/5}(j - \epsilon^{2/5}c_0 t)) + \epsilon^{1/3}\phi_P^\epsilon(\epsilon^{2/5}(j - \epsilon^{2/5}c_0 t)),$$
$$R_j(t) = \epsilon^{2/5}\phi_R^*(\epsilon^{2/5}(j - \epsilon^{2/5}c_0 t)) + \epsilon^{3/5}\phi_R^\epsilon(\epsilon^{2/5}(j - \epsilon^{2/5}c_0 t)),$$

solves the model.



Travelling Wave Problem

- The R_j-equation can be solved by direct integration and the P_j-equation by introducing an integrating factor.
- Substitution of the travelling wave ansatz into the A_j-equation results in a nonlocal differential equation.
- A Fourier transform and the introduction of a long wave scaling result into an implicit expression for the profile function.[4]
- By careful consideration of the formal long wave limit, a Bernoulli equation arises and the scaling relations unfold.
- Solving this equation yields expressions for the normalized leading order wave profiles ϕ_A^*, ϕ_P^* and ϕ_R^*.
- Invoking advanced perturbation theory, the main result follows.

References

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